

## LESSON 8

## Tessellations

## Math and 3D Modeling

In this lesson, you will learn:

- How to create four different types of tessellations, the tiling of a plane with no gaps or overlaps, using Fusion 360.
- How to implement offsets to get your 3D tiles to fit together.

For a **step-by-step guide** for the different types of tessellations, follow along with the Walkthrough sequence.

**Recommended age range 10+**

**Category:** Beginner lesson

**Tags:** 3D CAD, 3D printing, 3D printed, beginners, design, education, math, tiles

**Software:** Fusion 360

**Lesson Duration:** Three 45 minute classes (additional time to print)

**Estimated filament use (per person):** ~0.8m-3.0m/~7-21 g



## Lesson resources include:

- Lesson Overview
- Onesheet
- Walkthrough
  - Fusion 360
- Student CAD help sheet
- Example STL files
  - STLs
  - Code/CAD files

# Objectives

- Design and measure with precision
- Use Components
- Understand lines and splines
- Experiment, generate solutions, and solve problems
- Employ tolerances and offsets
- Prepare a model for 3D printing

# Background - Tessellations

A **tessellation**, is a tiling of the plane with no gaps or overlaps. The most common tessellations today are wall and floor tilings, using square, rectangular, hexagonal, or other shapes of ceramic tile. Other shapes can also be tessellated.

This project integrates math concepts for developing tiles that tessellate with the practical need of using offsets to get the 3D printed tiles to fit together. For this lesson, we will be using **Fusion 360**. (Though students are encouraged to apply these principles to other design software for further exploration.)

Tessellations can be created by performing one or more of three basic operations (**translation, rotation, and reflection**), on a [polyiamond](#).

# Types of tessellations

## Reflection

Mirroring the tile in the plane, as if being viewed in a looking glass.

## Rotation

Pivoting the tile in the plane. The rotation operation can be applied to any **polyiamonds** that do not possess circular symmetry. For example, the hexagonal **hexiamond**, which remains unchanged following rotation through  $60^\circ$  or multiples thereof.

## Translation

Sliding the the tile along the plane.

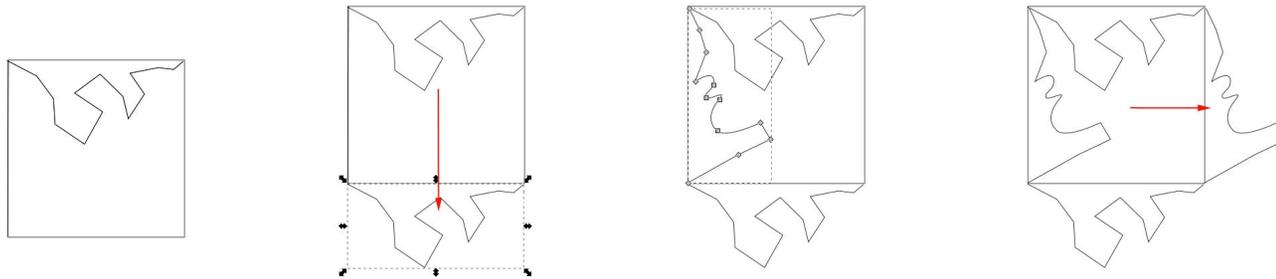
# Types of tessellations

**Simple tessellations** are those in which only the translation operation is used.

**Complex tessellations** are those in which one or both of the rotation and reflection operations is used with the translation operation. For example, a **glide reflection** tessellation is a combination of a **reflection** and a **translation**.

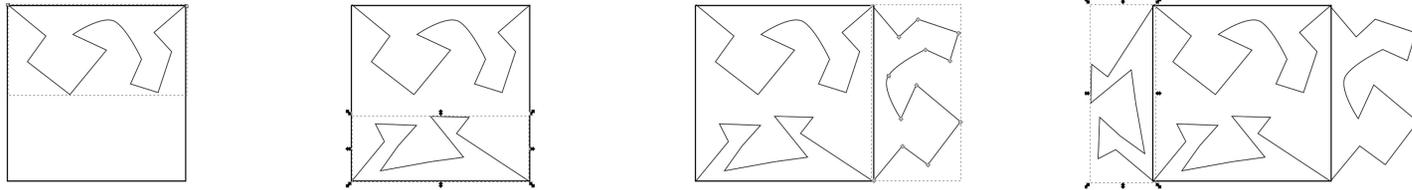
# Translation

1. Take a nibble from the top and translate it down.
2. Take a nibble from the left and translate it to the right.



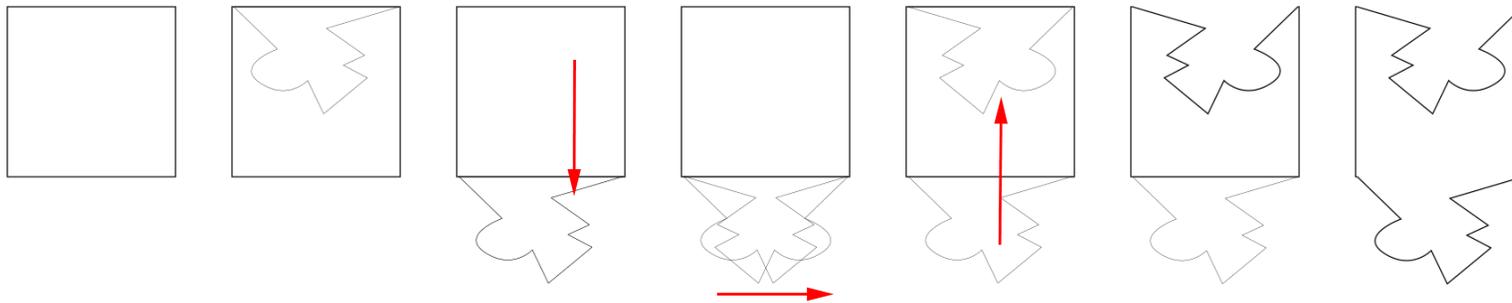
# Rotation

1. Take a nibble from the top (constrain the nibble to the top half) and translate and rotate it CCW so that it fits along the right side.
2. Take a nibble from the bottom and translate and rotate it CCW so that it fits along the left side:



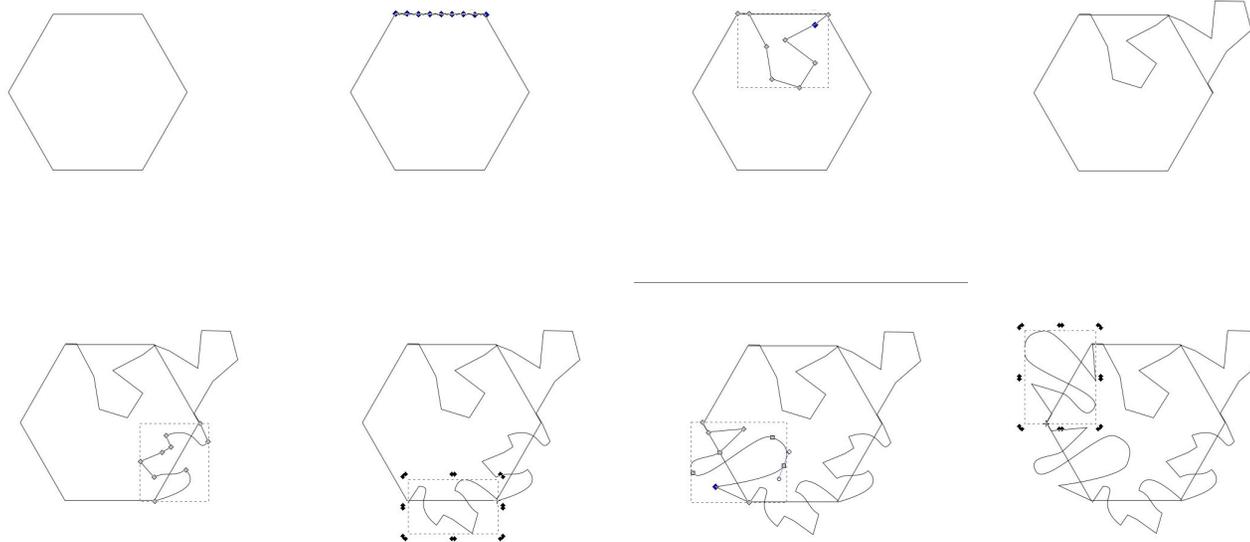
# Glide Reflection

1. Take a nibble from the top and translate it down.
2. Make a duplicate.
3. Flip the duplicated nibble over the horizontal axis.
4. Move back to top:



# A More Challenging Rotation

1. Start with a hexagon.
2. Take a nibble from a side and translate and rotate it CCW so that it fits along the edge to the right
3. Continue until all sides are covered:



# Project Advice

There is not one right way to do this project. It can be accomplished using a combination of commands. The goal is for you to break down the problem and use what might be the best series of commands to achieve your goal.

Once you have decided on the type of tile to construct, you'll need to create an offset. Why do you need an offset? **Remember:** A cylinder will not fit into a hole if they both have the same diameter.

If you want a cylinder to fit snugly into a hole make the diameter of the hole 0.15 mm- 0.2 mm larger. If you want the cylinder to slide, make the hole at least .5 mm larger. The offsets are known as tolerances and are determined by your printer.

When printing your tiles **do not** use a brim!

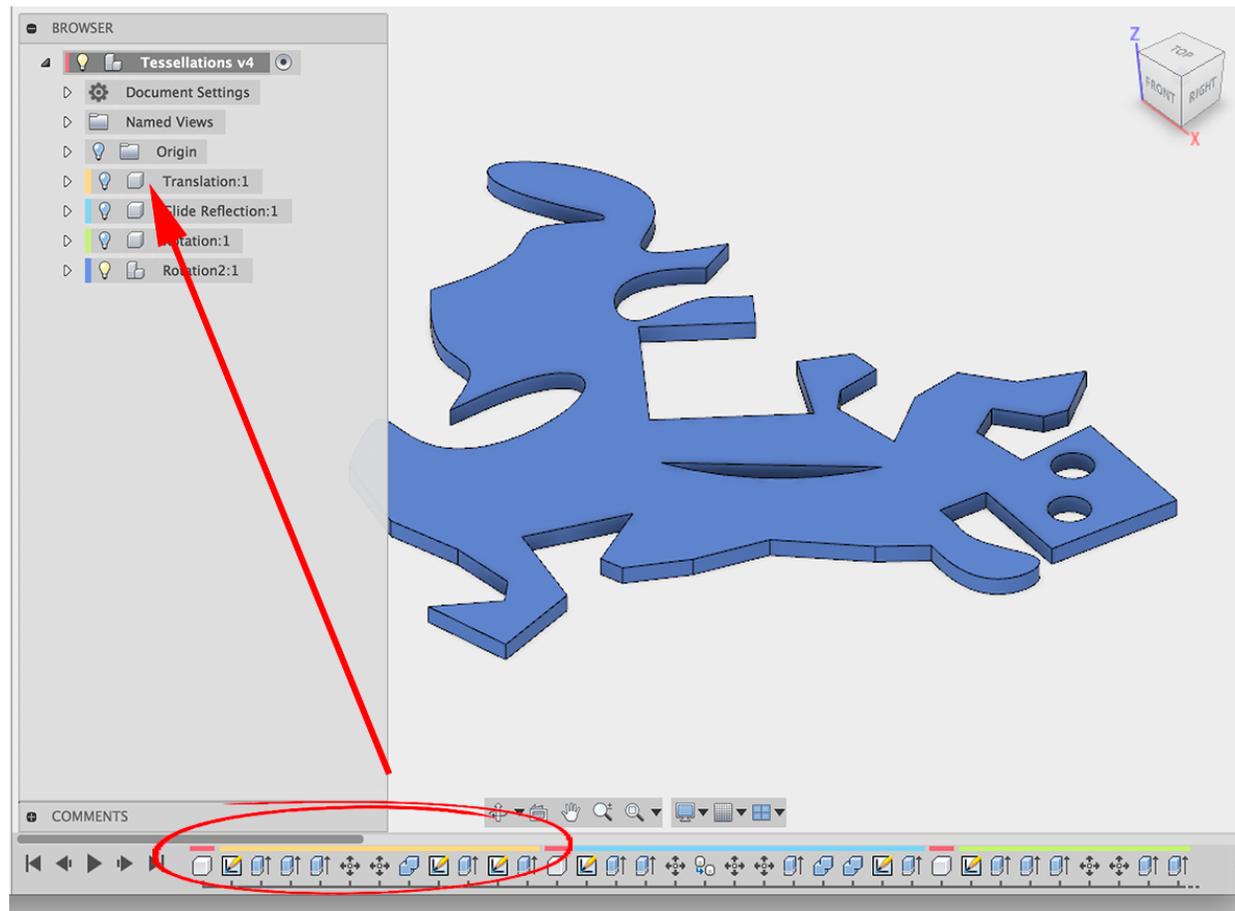
# Understand and incorporate components

A Fusion 360 component is a part with its own position and timeline.

- Component can contain bodies as well as other components.
- Components have their own origin planes whereas bodies do not.
- Components have their own timelines.
- Create Components at the start of your design.
- Activate a component to work within it.

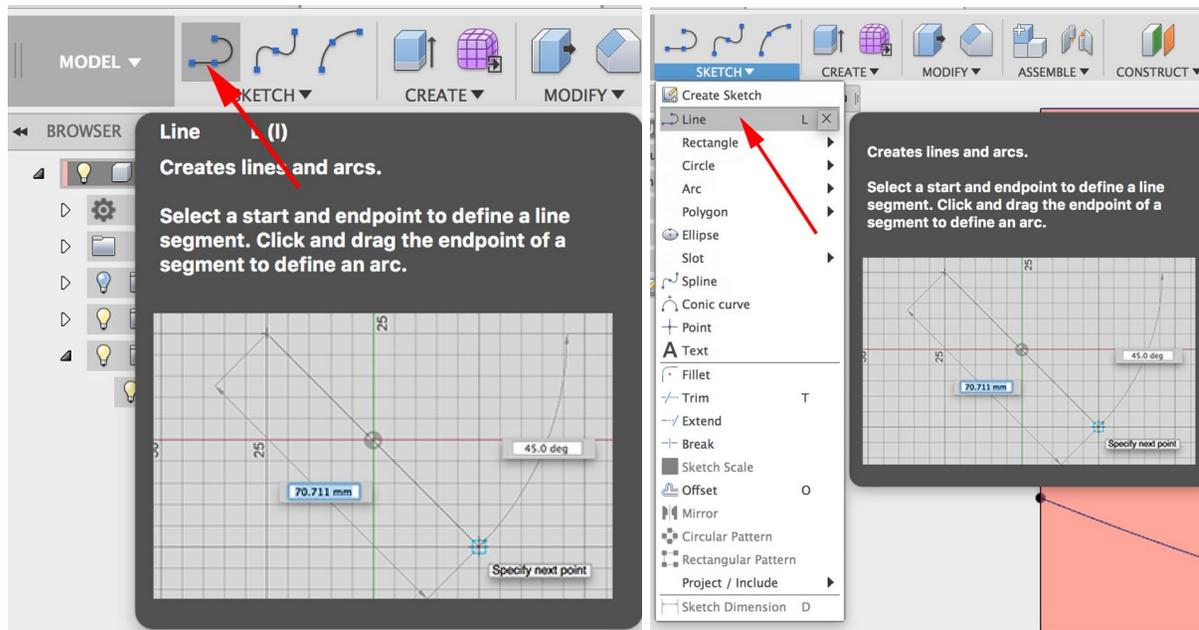
# Component Color Cycling

Turn Component Color Cycling on. Component color cycling is nice and easy way to tell which features in the timeline belong to which component.



# Lines

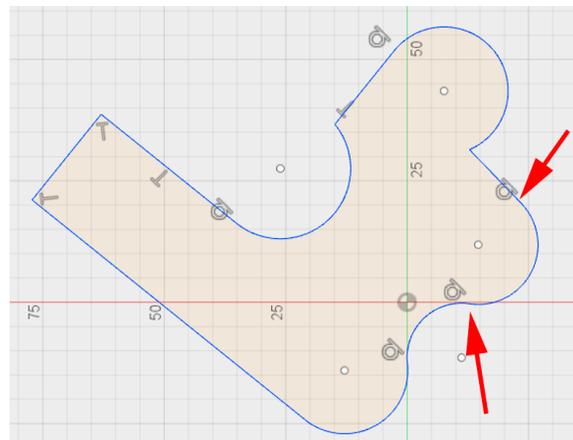
Lines can be drawn between any two points on the sketch plane, creating any length or angle needed. The line tool can be accessed in a number of ways. It is in the toolbar at the top, in the **Sketch** menu which, and can be activated by pressing the **L** key. It can also be activated in the right-click menu, under **Sketch** or in the **Marking** Menu at the bottom. This corresponds to the right-click gesture going straight down.



# Lines

With the line tool active, draw a line by clicking two endpoints. Clicking a second time on screen completes the line. The line tool is still active, and you can continue drawing lines. Fusion 360 keeps the line tool active so that you can create a chain of lines connected together. When you want to end the chain, click on the check mark shown nearby the line, or press the Escape key.

To create straight segments, just click on two endpoints. If you click on one endpoint and keep the mouse depressed while you move to another position and then release, you will get a curve:



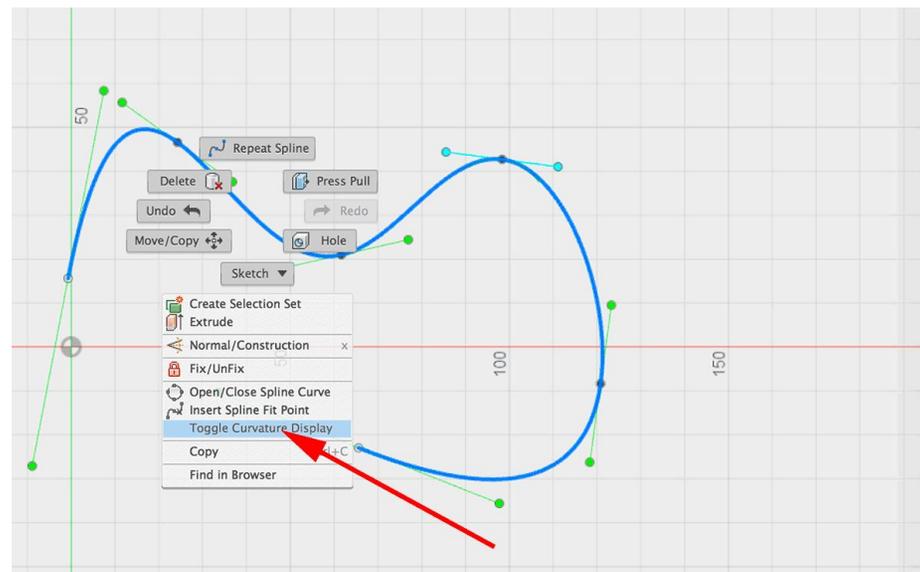
# Splines

The **Spline** command is found in the sketch menu and is used to create organic curves between points. To create a spline, enable the command, and then specify key points along the curve. As you place each point, notice the solid green line that shows the current spline between the specified point. To stop the spline, click on the green checkmark, and then press ESC to deactivate the spline commands. You can change the shape of a spline by clicking and dragging any of the spline points.

If you click on the spline itself to select it, notice the green tangent handles that appear at each spline point. These handles can be used to adjust the curvature of the spline at that point, also by clicking and dragging. You can also extend the length of the handle to widen the curve around that point. The changes you make affect the spline as a whole, not just this small section. However the spline point doesn't move, allowing for better control while manipulating the spline.

# Splines

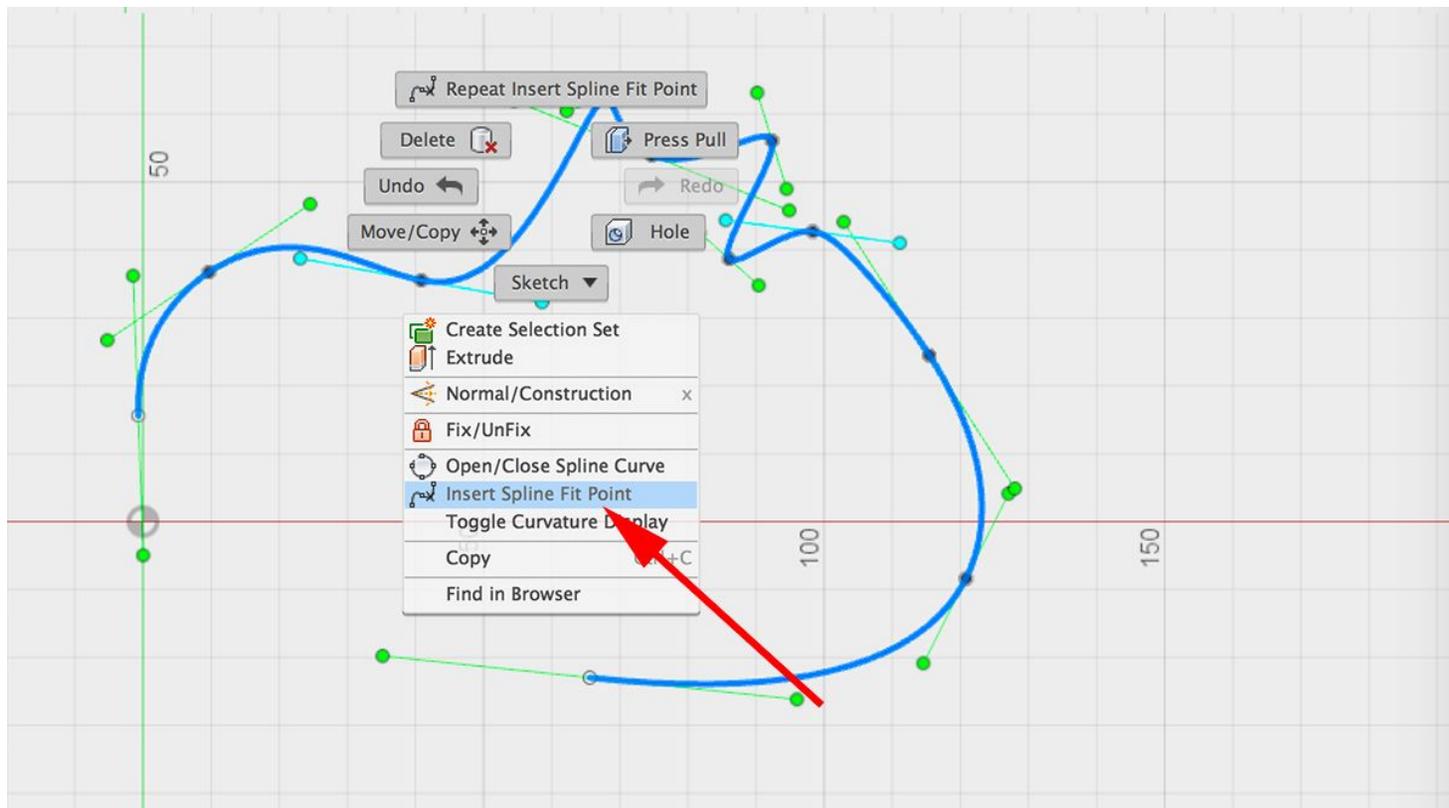
To help understand the spline's curvature, display the spline's **curvature comb**: a graphical representation of the curvature at each point, overlaid directly on the spline. To turn it on, I'll select the spline. Right click, and then select **Toggle Curvature Display** to turn the curvature comb on and off.



- Longer lines on the curvature comb indicate a larger rate of curvature, which also means there is a small radius at this point.
- If needed, the **comb density** and **scale** can be adjusted in the dialog box.

# Spline Fit Point

You can add more points to your spline by selecting the spline then right clicking. Then choose **Insert Spline Fit Point**. Click on the spline to add the point, then press ESC. Now you can edit the spline.



# Editing splines

You can also continue to edit the spline without editing the sketch.  
Just click and drag the points:

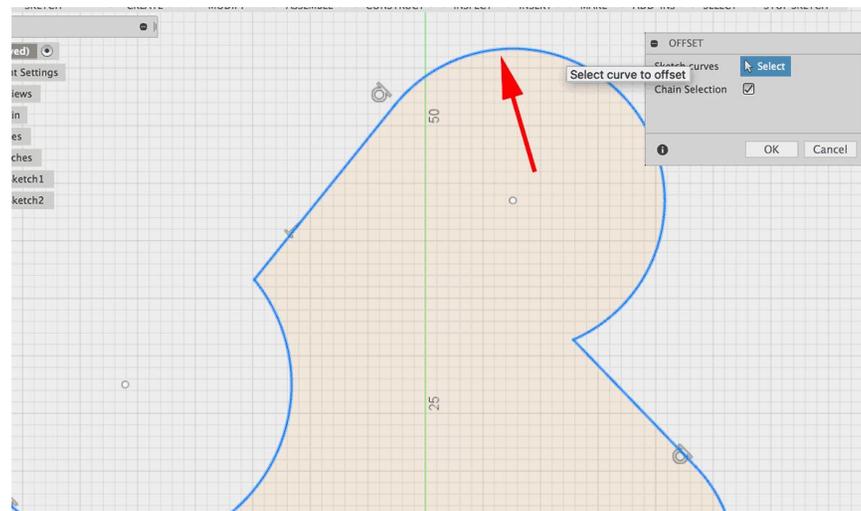


# Employ tolerances and offsets

To enable **Offset**, select it in the Sketch menu or press **O**.

Offset allows you to create an object at a specified distance from an existing object. After you offset objects, you can trim and extend them as an efficient method to create drawings containing many parallel lines and curves.

One thing to keep in mind is that an offset is dependent on the original object. You cannot create an offset of an offset, but must create your second offset based on the original object:



# Experiment, generate solutions, solve problems

The challenge of this assignment is to use the knowledge you have about **Fusion 360** (lines, splines, extruding, moving, and copying) along with the tessellation animations, to create your own tiles.

Study the animations and create a plan.

# Prepare models for 3D printing

When you test your models, print **two** copies. Do not use a brim. After you confirm that the tiles fit together, print a set of tiles.

In **Ultimaker Cura**, type **Ctrl/Cmd+M**, or **right click** on your model and select **Multiply Selected Model**. A dialog box will appear. Enter the total number of copies you'd like to make. In this case, type **1** and click **okay**.

